Code performance in multi-rate CDMA for an optical fiber network

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Abstract: We have developed analytical expressions for the crosscorrelation between users at unequal bit rates (chip rates are equal), i.e. for multi-rate (multi-processing gain) CDMA. These expressions for the crosscorrelations were used to generate data for Gold and Kasami sequences. The results of our calculations show that Gold and Kasami sequences' crosscorrelation performance in a multi-rate scheme is very close to the performance calculated when assuming random sequences.

Résumé: Nous avons développé des expressions analytiques pour l'inter-corrélation entre des usagers transmettant à des taux binaires différents (même taux de "chip"), c'est-à-dire un système à gains CDMA multiples. Ces expressions ont été utilisées pour générer une banque de données pour des séquences de Gold et de Kasami. Les résultats obtenus démontrent que la performance (inter-corrélation) des séquences de Gold et de Kasami dans un système CDMA à taux multiples est très proche de la performance obtenue en supposant des séquences aléatoires.

Local area networks (LAN) currently provide at most 100Mb/s of aggregate data transmission while future video-intensive applications are expected to drive requirements for bit rates up to 100Mb/s per user. This demand for greater bandwidth requires the use of optical fibers. In order to mitigate the costs of these new high bandwidth LANs, we can offer various levels of service, some high bit rate, some low bit rate (lower cost receivers). This means that future networks must have enough flexibility to accommodate users that do not require the maximum performance. Therefore there is a need for a multi-access scheme to provide multi-rate service.

We propose a multi-rate CDMA (code division multiple access) scheme for a high bandwidth LAN with the capacity to support 127 subscribers on the same wavelength, with as many as 10 active users transmitting simultaneously at 100Mb/s per user with a bit error rate (BER) of 10^{-9} using spreading codes of length 127. The multi-rate scheme that we consider is a multi-processing gain system (constant chip rate) with spreading codes of length 127, 255 and 511 that will provide data rates of 100Mb/s, 50Mb/s and 25Mb/s respectively. The transmitter power for each data rate will be different so that the energy per bit (E_b) remains constant [6].In this work we evaluate the performance of deterministic pseudorandom code sequences (Gold, Kasami) with respect to the Signal to Noise + Interference ratio (SNIR) in a given network. The parameter of interest is the average interference to be used in a preliminary system design analysis.

In this network, we use BPSK modulation with bipolar codes for DS/CDMA and homodyne coherent detection. This system supports *p* different data rates or subsystems and all users transmit asynchronously. The expression for the transmitted signal of user *k* in subsystem *i* is

$$s_{ik}(t) = \sqrt{2P_i}\,b_{ik}(t)a_{ik}(t)\cos(\omega_c t + \theta_{ik}) \tag{1}$$

where P_i is the average power of each user in the subsystem, ω_c is the carrier frequency, θ_{ik} is the phase, $b_{ik}(t)$ represents the data and $a_{ik}(t)$ is the periodic spreading sequence of period N_i. Both the bits and the chips are rectangular pulse shapes with the chip pulse duration T_c and bit pulse duration $T_i = N_i T_c$; and the amplitude of the data and spreading sequences take values from the set $\{\pm 1\}$.

Both the channel and the interference are modeled as additive white Gaussian noise (AWGN) [7]. The received signal is

$$r(t) = n(t) + \sum_{i=1}^{p}\sum_{k=1}^{K_i} s_{ik}(t-\tau_{ik}) \tag{2}$$

where $n(t)$ is the zero mean white Gaussian noise with double-sided spectral density $N_0/2$, K_i is the number of simultaneous users in subsystem *i* and τ_{ik} is the delay of signal $s_{ik}(t)$. The relative initial phase offsets θ_{ik} and the relative delays τ_{ik} are modeled as independent random variables uniformly distributed over $[0, 2\pi)$ and $[0, T_i)$, respectively.

In order to establish performance criteria we used the results obtained in [1] which give us the following expression for the SNIR of our network for a single-rate (i.e. $p = 1$) system

$$SNIR_{sr} = \frac{1}{\frac{(K-1)}{3N} + \frac{1}{SNR}} \tag{3}$$

where *SNR* is the single user Signal to Noise Ratio, *K* is the number of active users and *N* is the codelength. The interference term

$$\frac{(K-1)}{3N} \tag{4}$$

was derived using the correlation properties of random code sequences. The performance is evaluated through their respective periodic, aperiodic and continuous partial correlation properties [2],[3],[4].

The performance of Gold and Kasami sequences has already been evaluated [5], but under the assumption that all the sequences have the same codelength (period). In order to evaluate the performance of the multi-rate CDMA it is necessary to modify the analytical expressions for the cross-correlation among different rate users. The new expression for the interference term will permit the evaluation of performance as a function of the sequence length. The output of the receiver (matched correlator) *l* in subsystem *j* is

$$Z_{jl} = \int_0^{T_j} r(t) a_{jl}(t) \cos(w_c t) dt \tag{5}$$

where $a_{jl}(t)$ is the despreading sequence, corresponding to user *jl*. This output consists of the desired signal + Mutiple Access Interference (MAI) + AWGN. Next, we evaluate the variance, $\mathrm{var}[Z_{jl}]$, focussing on the variance of the MAI at the output of the receiver. The variance of the MAI is

$$\mathrm{var}[I_{jl}] = \sum_{\substack{i=1 \\ (i \neq j,\, k \neq l)}}^{p} \sum_{k=1}^{K_i} \frac{1}{2T_j^2} \frac{P_i}{P_j} E_{b_{ik}, \tau_{ik}}[J_{ik}{}^2] \tag{6}$$

where T_j is the desired signal's pulse interval, P_j is the desired signal's received average power, P_i is the interfering signal's average power and

$$J_{ik} = \int_0^{T_j} b_{ik}(t - \tau_{ik}) a_{ik}(t - \tau_{ik}) a_{jl}(t) dt \tag{7}$$

To evaluate $\mathrm{var}[I_{jl}]$, we must average the term J_{ik}^2 over the interfering data bits and the relative delays. We find

$$E_{b_{ik}, \tau_{ik}}[(J_{ik})^2] = \frac{T_c^3}{3T_i} \cdot r_{ik} \tag{8}$$

where r_{ik} is called the average interference parameter created by user *k* in subsystem *i*. This parameter is a function of the aperiodic crosscorrelation and is derived as described in [3], but modified to take into account the multirate aspect. The expression for r_{ik} must be evaluated for three different cases: $T_j = T_i$, $T_j > T_i$ and $T_j < T_i$ where the first case is the single-rate result. The relation between the sequence lengths is

$$N_y = LN_x + (L - 1) \tag{9}$$

where $L = 2^{(n_y - n_x)}$ and where $n_y \geq n_x$ are the degrees of the sequences.

This network requires a power control scheme that keeps the average energy per bit received constant at all transmission rates. This means

$$E_b = P_i T_i = P_j T_j \tag{10}$$

The simulations were carried out using Gold sequences of length $N = 127$, $N = 511$ and Kasami sequences of length $N = 255$. There are N sequences of each length. The results were obtained by calculating the average interference parameter r_{ik} of each sequence of a given set of sequences with all N_i sequences of one of the two other lengths. The $N_j \times N_i$ values of r_{ik} were plotted as an histogram. The simulations were repeated to cover all three cases, namely $T_j = T_i$, $T_j > T_i$ and $T_j < T_i$.

The results obtained in the single-rate case (see Fig. 1) are consistent with those published in [4],[5] where the value of the average crosscorrelation parameter is approximately $2N^2$. In Fig. 1, the distribution of r_{ik} is normalized over the value obtained with the random sequence assumption ($2N_j^2 = 522242$ for $N_j = 511$). If we substitute this value in eq. 8 we obtain

$$E_{b_{ik}, \tau_{ik}}[(J_{ik})^2] = \frac{T_c^3}{3T_i} \cdot 2N_i^2 = \frac{2N_i^2}{T_i} \cdot \frac{T_c^3}{3} \tag{11}$$

In the case where $T_j > T_i$ the simulation results (see Fig. 2 and 3) are consistent with those published in [6] where $(r_{ik} = 2LN_i^2)$. Again, the distribution of r_{ik} in Fig. 2 and 3 is normalized over the respective values obtained with the random sequence assumption. If we substitute these results in eq. 8 we obtain

$$E_{b_{ik}, \tau_{ik}}[(J_{ik})^2] = \frac{T_c^3}{3T_i} \cdot 2LN_i^2 = \frac{2LN_i^2}{T_i} \cdot \frac{T_c^3}{3} \tag{12}$$

Lastly, in the case where $T_j < T_i$, the simulation results (see Fig. 4 and 5) are also consistent with [6] where $r_{ik} = 2N_jN_i$. Fig. 4 and 5 show the distribution of r_{ik} normalized over the values obtained with the random sequence assumption. If we substitute the results for the average value of r_{ik} into eq. 8 we obtain

$$E_{b_{ik}, \tau_{ik}}[(J_{ik})^2] = \frac{T_c^3}{3T_i} \cdot 2N_jN_i = \frac{2N_jN_i}{T_i} \cdot \frac{T_c^3}{3} \tag{13}$$

With the approximation that $LN_i / N_j \cong 1$ and in the case where $T_j > T_i$, using the average value for r_{ik} in eq. 6, we have

$$\mathrm{var}[I_{jl}] = \frac{1}{3N_j} \sum_{\substack{i = 1 \\ (i \neq j, k \neq l)}}^{p} \frac{P_i}{P_j} K_i - 1 \tag{14}$$

This result matches that found when modeling the codes as random sequences. The multi-rate network performance, in terms of simultaneous users, can thus be express as follows:

$$SNIR_{\text{multirate}} = \frac{1}{\frac{1}{3N_j}\sum_{i=1}^{p}\frac{P_i}{P_j}K_i - 1 + \frac{1}{SNR}} \qquad (15)$$

With these results we conclude that the random sequence assumption [4],[6] for preliminary system design is a good one. The second conclusion is that the characteristics that make Gold and Kasami sequences interesting in a single-rate CDMA network are also valid in a multi-rate CDMA network.

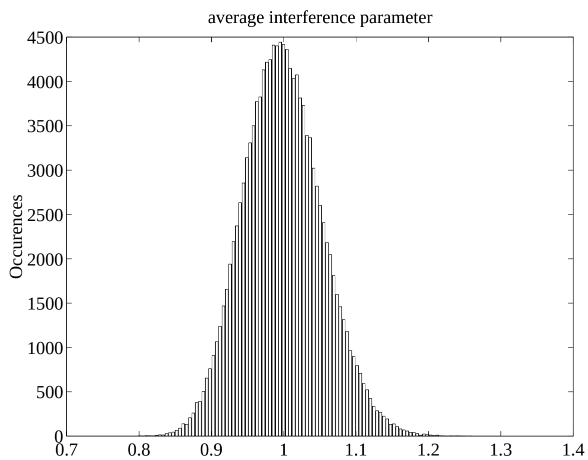

Figure 1: Single-rate crosscorrelation with sequence length $N_j = N_i = 511$.

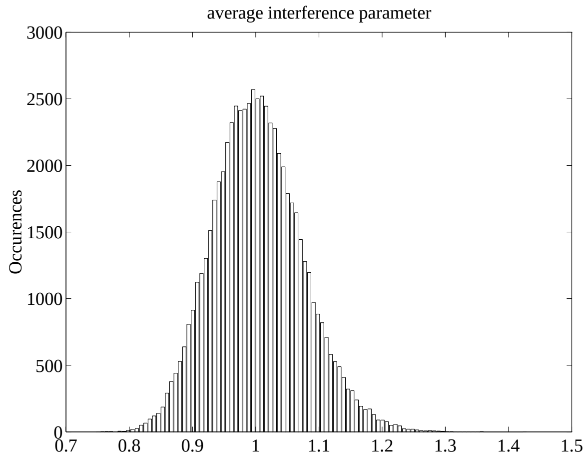

Figure 2: Multi-rate crosscorrelation with "signal" sequence length $N_j = 511$ and interfering sequence length $N_i = 127$.

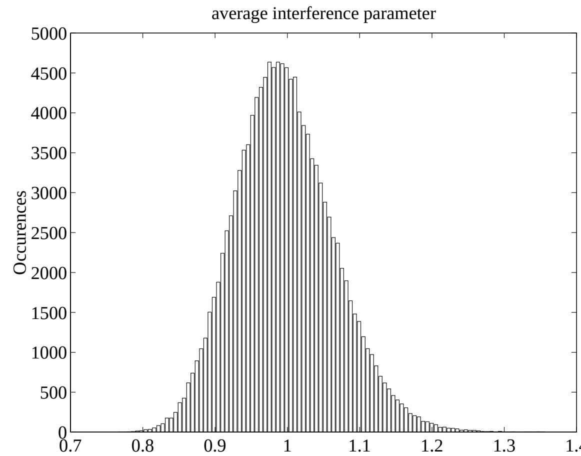

Figure 3: Multi-rate crosscorrelation with "signal" sequence length $N = 511$ and interfering sequence length $N = 255$ (Kasami sequences from large set).

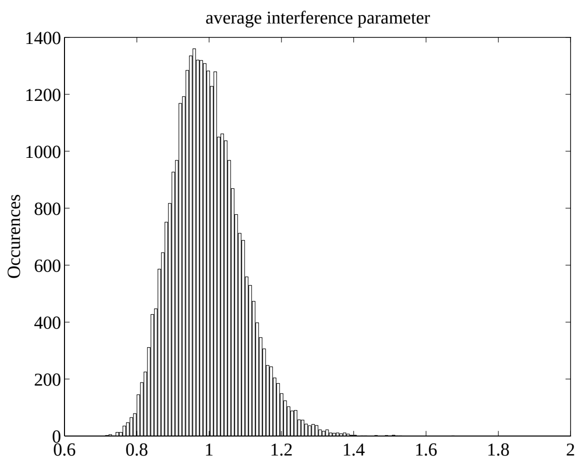

Figure 4: Multi-rate crosscorrelation with "signal" sequence length $N_j = 127$ and interfering sequence length $N_i = 255$.

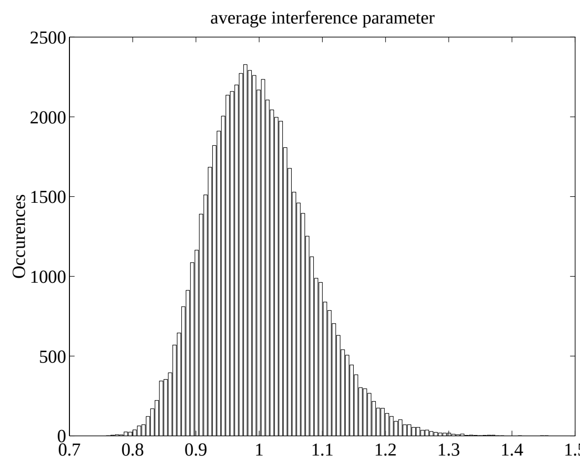

Figure 5: Multi-rate crosscorrelation with "signal" sequence length $N_j = 127$ and interfering sequence length $N_i = 511$.

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