CQPSK performance over frequency-selective fading channels by the Gaussian approximation and the Fourier-Bessel method.

A.Semmar, H.T.Huynh and M. Lecours, *Fellow, IEEE*
Dept. of Electrical and Computer Eng.
Université Laval
Québec, Qué, Canada, G1K 7P4

Abstract: *The purpose of this paper is to compare the Gaussian approximation and the Fourier-Bessel series method for computing the CQPSK performance over frequency selective Rician fading channels described by a one-sided exponential delay profile. Using Gaussian approximation, we obtain the bit error rate (BER) in terms of a Fading Factor. With the Fourier-Bessel method, we derive an upper bound for the BER by assuming that all path strengths are identically Rayleigh distributed. Comparisons of the results obtained in the two cases permit to investigate when the Gaussian approximation approach is applicable and when the use of another method such as Fourier-Bessel series is required. The performance resultats based on Monte Carlo simulations show the tightness of the upper bound for the BER obtained with the Fourier-Bessel method.*

1. INTRODUCTION

The calculation of the error performance of digital transmission systems in the presence of intersymbol interference and noise is a longstanding problem on which considerable attention has been focused [1], [2], [3]. For broadband PCS systems with high data rates, major impairments are due to intersymbol interference (*ISI*) and to the delay spread which impose an upper limit on the data rate and degrade system performance.

This work is concerned with a comparative study of the performance of CQPSK communications over Rician fading channels, as evaluated by using the Gaussian approximation and the Fourier-Bessel series method. We consider that the power delay profile is one-sided exponential model.

Previous analyses of BFSK and BPSK [4], [5] systems have studied the average probability of error in frequency-selective fading channels by using a zero-mean Gaussian channel model. This assumption remains valid only if the number of *ISI* components is large and if they are independant and identically distributed, so that the central limit theorem can be considered. For this case, we develop an analytical technique for the evaluation of the *BER* of CQPSK communications in terms of a *Fading Factor* taking into account the effect of the time delay spread.

For other cases, when the number of *ISI* components is small and when their probability density function cannot be expressed in a closed form, it is necessary to compute the *BER* by numerical methods.

Various methods have been proposed to compute the error probability in communications engineering when the channel is not of the AWGN type. Shimbo and Celebiler [1] and Prabhu [2] have shown respectively how the error probability can be expressed as a Gram-Charlier series and as an Hermite series, but their computations are extremely involved. Tighter upper and lower bounds have been obtained by Matthews [3] as function of the interference variance, but these bounds become less effective for large signal-to-noise ratios.

In this paper we have adopted the Fourier-Bessel series method developed by Bird [6] to determine an upper bound for the error probability. This method is easy to formulate mathematically, gives a very low calculation error and converges quickly even for high values of the signal-to-noise ratio (*SNR*) [6], [7]. This approach was used by Bird to calculate the probability of error for CPSK and ASK communications systems affected by Gaussian noise and cochannel interference [6]. It has also been applied to QAM sytems in the presence of an additive Gaussian and impulsive noise [7]. This method is based on the knowledge of the characteristic function of the *ISI* components. The single hypothesis required is that the *ISI* components are independant random variables.

The bit error rate for the system was also evaluated through Monte Carlo simulations in order to verify the validity of the upper bound developped with the Fourier-Bessel method.

A brief outline of the paper is as follows. Section 2 describes the channel and the power delay profile models. Details of the *BER* performance evaluation using the Gaussian approximation and the Fourier-Bessel series method and a desciption of the simulation technique are given in section 3. The numerical results are presented in section 4. Section 5 provides the conclusions.

2. SYSTEM MODEL

The channel model used in this study is similar to the model defined in [5] where the transmitted waveform is propagated through a noisy fading multipath channel. The multipath comprises N fading paths. Each path n ($n = 0, 1, 2..N-1$) is described by a strength coefficient r_n, a time delay τ_n and a carrier phase shift ϕ_n introduced by the path. The receiver used here is the classic coherent quaternary phase-shift keying receiver. The received signal $r(t)$ can be written as [8]:

$$r(t) = \sum_{n=0}^{N} \sum_{i} \sqrt{\frac{2E_s}{T_s}} \cdot r_{n,i} V(t - iT_s - \tau_{n,i}) \cos(2\pi f_c t + \theta_i + \phi_n) + n(t) \tag{2}$$

where E_S is the energy per symbol, T_S is the symbol duration, $V(t)$ is a unit amplitude rectangular pulse of duration T_S, f_c is the carrier frequency and θ_i is the carrier phase. The noise signal is assumed to be an additive, zero-mean, white Gaussian noise (AWGN) statistically independant of the multipath with spectral density $N_0/2$. The carrier phase shift ϕ_n introduced by the path is uniformly distributed over $[0,2\pi]$, the number of echoes follows a Poisson distribution and their arrival time is uniformly distributed [4].

For the Rician channel an *LOS* signal is present, the received signal consists of: the desired signal, the intersymbol interference and the additive Gaussian noise. We assume, as usually done, that the intersymbol interference affects only adjacent data pulses, this condition being satisfied if $p(\tau) \approx 0$ for $\tau > T_S$. Under this assumption, the statistics of $r(t)$ depend on two consecutive data symbols $[S_{i-1}, S_i]$. A received symbol is then affected by the first part of its own interference I_i^X and the second part of the interference generated by the previous symbol I_{i-1}^X [8]:

$$\begin{aligned} I_i^X(i) &= \sum_{n=1}^{N} \sqrt{E_s} \cdot r_X^i(\tau_n) \cdot \left(1 - \frac{\tau_n}{T_s}\right) \cdot \cos(\theta_i + \phi_n) \\ I_{i-1}^X(i) &= \sum_{n=1}^{N} \sqrt{E_s} \cdot r_X^{i-1}(\tau_n) \cdot \left(1 + \frac{\tau_n}{T_s}\right) \cdot \cos(\theta_{i-1} + \phi_n) \end{aligned} \tag{3}$$

The power delay profile model used is the exponential profile [4] given by:

$$p(\tau) = \alpha \cdot \exp\left(-\frac{\tau}{D}\right) \qquad \tau \geq 0 \tag{4}$$

Parameter α describes the power level implied in the delay spread model relative to the power level of the main signal component. D is the root-mean-squared (rms) delay spread. We define the power delay profile as the average of the echoes arriving with time delay τ: $p(\tau) = E\{r^2(\tau)\}$ [9] .

3. EVALUATION OF THE BIT ERROR RATE

3.1 Gaussian approximation:

In the Gaussian approximation method, the evaluation of the probability of error per bit assumes that the total effect of all echoes is approximated by a Gaussian random variable [4], [5]. For analyzing the *BER* performance of QPSK systems in this case, we consider that the in-phase and the quadrature components X and Y outputs are uncorrelated zero-mean variables with the same variance: $\sigma_X^2 = \sigma_Y^2$. The *BER* performance of the system is evaluated by assuming that the total effect of all significant echoes can be represented by a Gaussian process [9]:

$$P_e \approx erfc\sqrt{\frac{\gamma}{1 + F\gamma}} \tag{5}$$

where $\gamma = E_b/N_0$ represents the signal-to-noise ratio, E_b is the energy per bit and F is a *Fading Factor* defined as: $F = 2\sigma_X^2/E_b$. In the calculations of the *Fading Factor*, we have taken into account the power delay profile, the average number of echoes $\nu = E\{N\}$ arriving in the detection interval, the parameter α and the normalized root-mean-squared delay $\mu = 1/\xi$ to obtain the following expression [9]:

$$F = \frac{2\alpha\nu}{\xi^3}[(\xi - 1)^2 + 3 - e^{-\xi}\{(\xi + 1)^2 + 3\}] \tag{6}$$

3.2 Fourier-Bessel method:

We derive in this section an expression for the bit error rate performance of CQPSK systems using the Fourier-Bessel series expansion method developed by Bird [6]. This method has been shown to behave well and to converge quickly even for high values of the signal-to-noise ratio [6], [7].

Our assumptions for the Gaussian process are also valid for this method except that the intersymbol interference is considered as a sum of independant variables and that there is no closed form expression for its probability density function. But on the other hand its characteristic function is known and can be simply expressed as the product of the characteristic function of each variable. The bit error rate in this case is bounded by the distribution function of the receiver in-phase or quadrature signal:

$$P_e = P(X < 0, Y < 0 | \theta_i = \pi / 4) \leq 2P(X < 0) = P_0 \quad (7)$$

The problem is now reduced to computing P(X < 0). Using the methodology presented by Bird [6], the expression for the upper bound of the error probability is defined by:

$$P_0 = 1 - \frac{4}{\pi}\left(\sum_{m=1}^{M} \frac{\Phi(B_m) \bullet \sin(B_m \cdot d)}{2m - 1} \right) \quad (8)$$

where $B_m = (2m-1)\pi/2l$, $\Phi = \Phi_I \cdot \Phi_G$, Φ_I and Φ_G are the intersymbol interference and the normalized Gaussian noise characteristic functions, and $2d$ is the minimum distance between two adjacent points in the *QPSK* constellation. l is the maximum value of the *ISI* random vector; it can be expressed as in Bird [6] by $l = 10\sqrt{\gamma}$. Φ_G is given by:

$$\Phi_G(B_m) = \exp(-B_m / (2\alpha\gamma)) \quad (9)$$

In the calculation of Φ_I, we take into account the power delay profile, the average number of echoes ν, the normalized root-mean-squared delay μ, the parameter α and the amplitude of each echo [9].

$$\Phi_I(B_m) = \exp\left[\nu\left(\sum_{k=1}^{K} (-B_m^2)^k \frac{(2k-1)!!}{(k!)^2} \left\{ \int_0^1 (x^{2k} + (1-x)^{2k}) e^{-k\frac{x}{\mu}} dx \right\} \right) \right] \quad (10)$$

3.3 Simulation technique:

Let β be the relative precision obtained from M simulations to estimate the mean error probability P_e. For a confidence of 0.99, one has $M \geq 3.96 \cdot (1/P_e - 1)/\beta^2$. For a precision less than 2% for $P_e = 10^{-2}$ and a precision of 19% for $P_e = 10^{-4}$, we used 10^6 simulations. This degree of precision is adequate considering the order of magnitude of the average error probability to estimate [8]. We have considered that the channel’s statistics are time-invariant over 10^4 symbol periods. This is a reasonable assumption for communications systems at high data rates.

4. NUMERICAL RESULTS

Figure 1 gives the bit error rate in function of E_b/N_0 for the Fourier-Bessel series and for the Gaussian approximation methods. The curves correspond to two sets of values, namely a first set ($\nu = 4$, $\mu = 0.5$, $\alpha = 0.3$) and a second set ($\nu = 2$, $\mu = 0.1$, $\alpha = 0.2$) where ν is the average number of echoes, μ is the normalized rms delay and where α is a parameter which describes the power level implied in the delay spread relative to that of the main signal component.

In the Gaussian approximation, the first set of values corresponds to a *Fading Factor F* in the order of 0.7, while the second set of values corresponds to *F* in the order of 0.07. For the specific case where the *Fading Factor* is equal to zero, the performance reduces to the classical result for *QPSK* modulation in Gaussian noise.

The first set of values illustrates cases where the average number of echoes $\nu = 4$ is relatively important and, with $\mu = 0.5$, one sees that echoes of a significant power level are likely to be present for the duration of the symbol. These characteristics confer these cases a somewhat gaussian-like quality. As a consequence, the *BER* in function of E_b/N_0 evaluated with the Gaussian approximation method are nearly identical to those given by the Fourier-Bessel method.

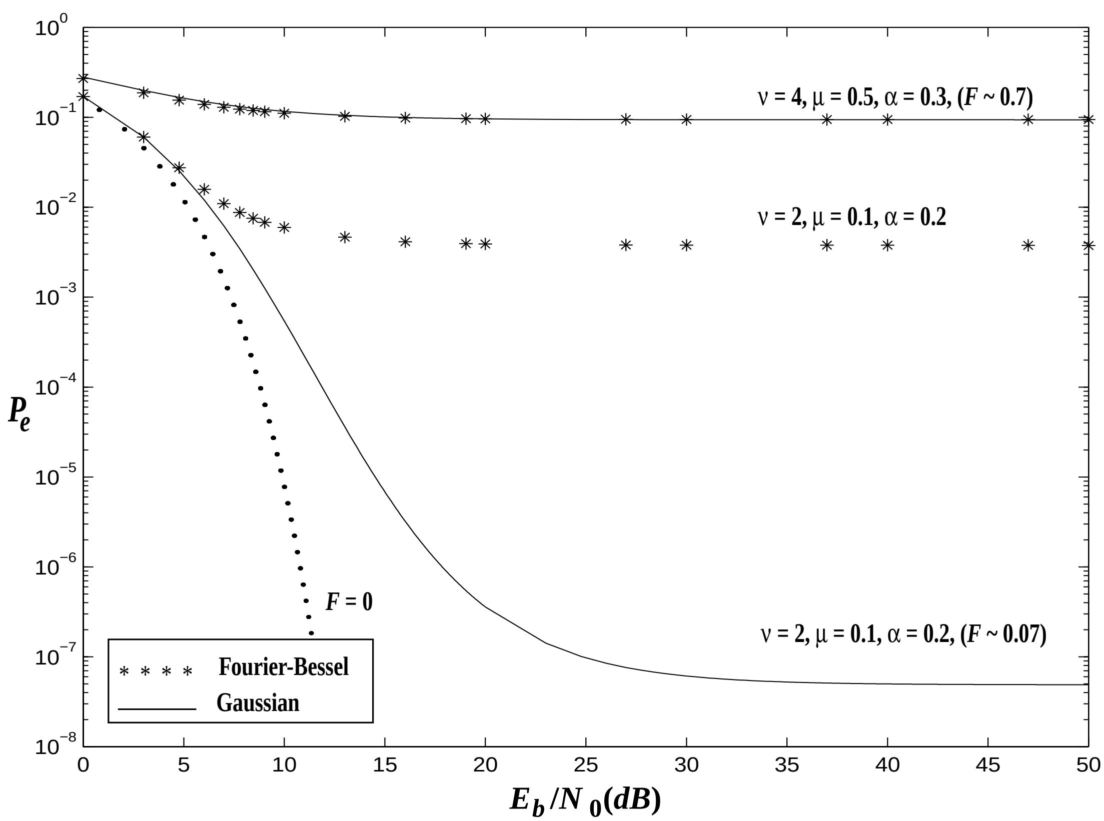

Figure 1 : Bit error rate for the Gaussian approximation and the Fourier-Bessel method in function of E_b/N_0.

The second set of values illustrates cases where the average number of echoes ν = 2 is relatively small; with μ = 0.1, these echoes are concentrated in the first part of the symbol.

Figure 1 shows that, in such cases, the Gaussian approximation strongly underestimates the degradation caused by the *ISI*, as compared to the Fourier-Bessel method.

In general, the curves on Figure 1 show that the effect of the intersymbol interference becomes dominant as the signal-to-noise ratio becomes larger and that one tends towards an irreducble error rate P_I. Figure 1 and other results obtained show that, as the normalized rms delay becomes smaller, the effect of the *ISI* decreases and the system performance improves. The system performance degrades as the average number of multipath components ν increases. The power level implied in the delay spread relative to the power level of the main signal component, which is described by α, has a strong influence on system performance.

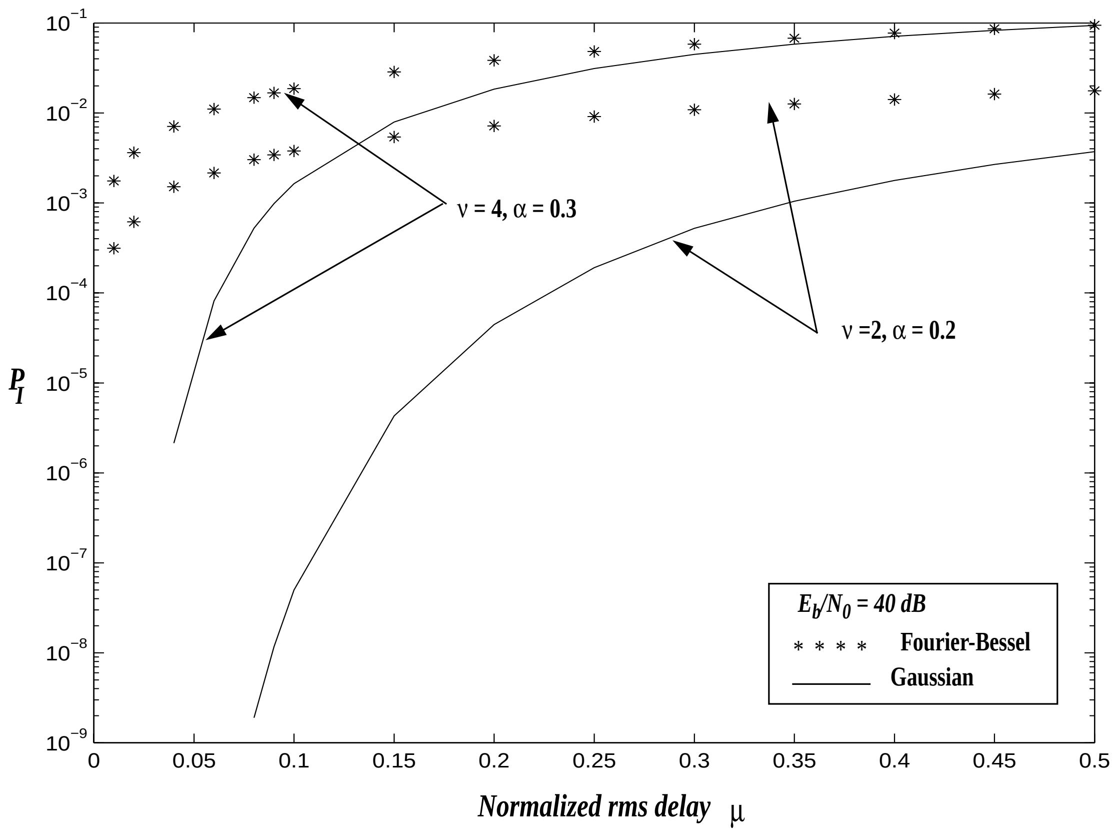

Figure 2 : Irreducible bit error rate with the Gaussian approximation and the Fourier-Bessel method in function of the μ.

Figure 2 shows the irreducible error probability per symbol P_I in function of the normalized rms delay μ. For pratical purposes P_I is taken here as the value of the probability of error when the E_b/N_0 ratio equals 40 *dB*. One notes that, for the same value of the normalized rms delay, the irreducible error probability strongly depends on the parameters ν and α and that, the smaller the value of the normalized rms delay spread μ, the better the system performance will be. It is observed on Figure 2 that the Gaussian approximation and the Fourier-Bessel curves for the case (ν = 4, α = 0.3) start to diverge when the *Fading Factor F* becomes smaller than 0.6: this is consistant with the observations on other cases.

In general, the results indicate that the Gaussian process and the Fourier-Bessel series method give the same performance estimates only for relatively high values of the normalized rms delay spread, of the average number of scattering paths and of the power level implied in the scattering paths relative to that of the main signal component.

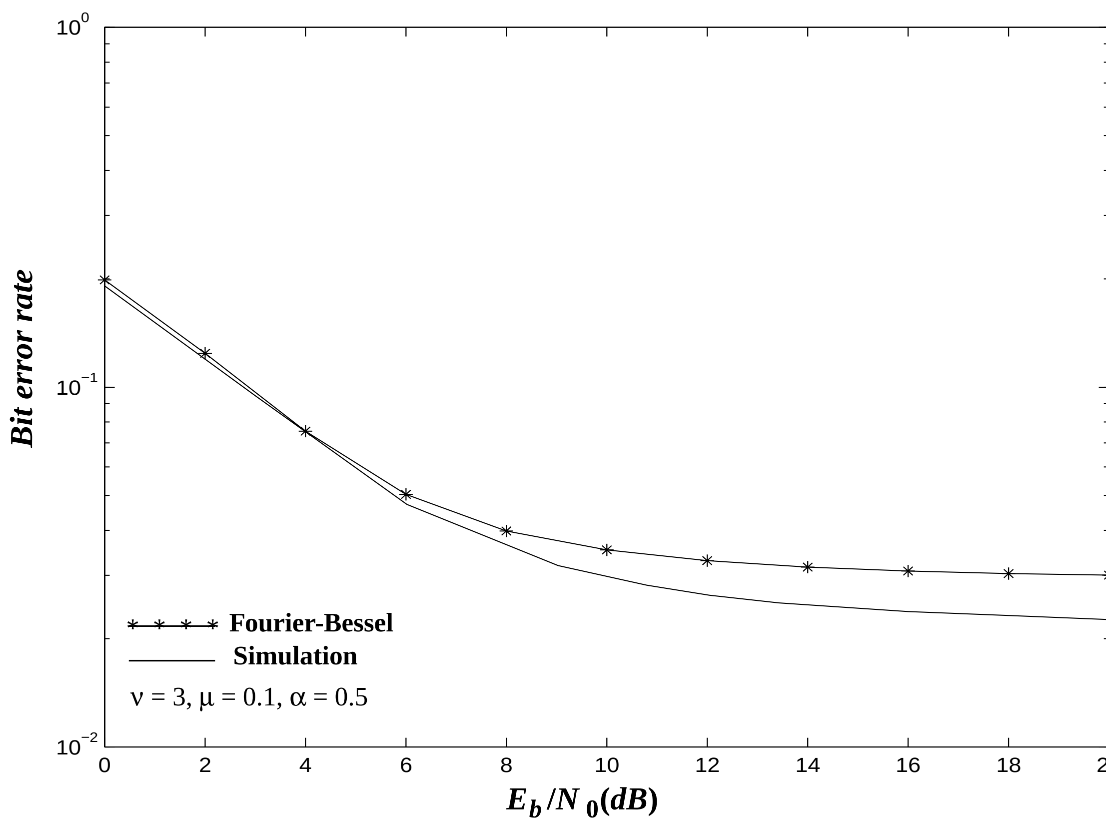

Figure 3 : Bit error rate for the Fourier-Bessel method and Monte Carlo simulation results in function of E_b/N_0.

Figure 3 represents the simulation results and the upper bound of the average probabilty of error P_e for QPSK system operating in multipath fading channels and Gaussian noise. The curves are for the parameter $\alpha = 0.5$, the normalized rms delay spread $\mu = 0.1$ and the Poisson parameter $\nu = 3$. Comparing with the simulation results, the figure indicate that the upper bound of the *BER* is tight for small signal-to-noise ratio (SNR < 8 dB) and that, for high SNR, the curves have a small divergence.

5. CONCLUSION:

In this study, the performance of coherent *QPSK* communication system has been analysed. Expressions for the symbol error rate are derived in the context of frequency selective Rician fading channel by using an exponential time delay spread model. The results for the bit error rate has been presented with two methods: Gaussian approximation and Fourier-Bessel-series.

Comparisons of the results obtained with the Fourier-Bessel series computations with those for the Gaussian approximation permit to conclude that the Gaussian approximation is useful for the performance analysis of systems operating in a multipath environment in certain cases, for instance when the *Fading Factor F* is larger than 0.6. For the other cases, the simple Gaussian approximation is not sufficient and it is necessary to proceed with the more involved Fourier-Bessel performance estimation method. The simulation results has confirmed the usefulness of the Fourier-Bessel approach. It is clear also that, in order for QPSK to be used for high data rate systems in multipath fading channels, it is necessary to consider using diversity techniques, equalization, adaptative antennas. The approaches for performance analysis presented here could be used in the design process of such complex systems.

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