# Performance of coherent QPSK communications over frequency-selective fading channels for broadband PCS.

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***Abstract***: *The fading caused by the multipath and the effect of delay spread affect broadband wireless personal communications channels by introducing intersymbol interference which appears when the root-mean-squared* (*rms*) *delay spread is significant in comparison to the transmitted symbols durations and results, without equalizing, in an irreducible error probability. In this paper, we show that it is easy to obtain, for given power delay profile models, the variance of the interference components at the output of a QPSK receiver. Using Gaussian approximations, a general form for a "Fading factor" is defined. This approach is applied, in the particular cases of exponential and Maxwell power delay profiles, to analyze the performance of coherent QPSK modulation over frequency selective fading channels.*

***Résumé***: *Les évanouissements causés par les trajets multiples et l'effet du profil de délai dans le canal affectent les systèmes de communications personnelles à large bande en introduisant de l'interférence intersymbole, qui se manifeste quand la valeur quadratique moyenne du profil de délai est significative en comparaison avec la durée des symboles, et qui résulte, en l'absence d'égalisation, en une probabilité d'erreur irréductible. Nous montrons dans cet article qu'il est facile d'obtenir, pour des modèles quelconques de profils de délai, la variance des composantes interférentes à la sortie d'un récepteur QPSK. En utilisant l'approximation gaussienne, nous définissons une forme générale pour un "Facteur de Fading". Nous appliquons cette approche, dans les cas particuliers de profils de délai exponentiels et de Maxwell, à l'analyse de la performance de la modulation QPSK sur les canaux à fading sélectif en fréquence.*

## I. INTRODUCTION:

This paper is concerned with the evaluation of the average symbol error probabilities for coherent *QPSK* communications over fading channels. This work is particularly pertinent for evaluating the performance of broadband wireless communication systems in a selective fading environment. Previous results have been obtained for the performance of *FSK* for the Gaussian, exponential, rectangular, one-sided exponential and Maxwell time delay spread models [1]-[2].

We study two classes of fading models, the class of Rayleigh selective channels for which we use a non-symmetrical Maxwell time delay spread and the class of Rician selective channels for which we consider that the delay spread is one-sided exponential. The received signal consists of three parts: the desired signal (*LOS* for Rice channels and the synchronized signal for Rayleigh channels), the scattering components and the channel noise. We assume that the two last parts can be approximated by independant, Gaussian, zero-mean random variables. The system performance is estimated by using their variances.

A brief outline of the paper is as follows. The model of the *QPSK* system is described in section II. Details of the symbol error rate performance evaluation and the expressions of the delay power profiles are given in section III. In section IV the numerical results are presented. Conclusions are given in section V.

## II. SYSTEM MODEL

The channel model used in this study is similar to the model defined in [3] and [4] where the transmitted waveform is propagated through a noisy fading multipath channel. The multipath comprises $N$ fading paths. Each path $n$ ($n$= 0, 1,.. , N-1) is described by a strengh coefficient $r_n$, a time delay $\tau_n$ and a carrier phase shift $\phi_n$ introduced by the path which we assume uniformly distributed over $[0,2\pi]$. The received signal can be written as:

$$r(t) = \sqrt{2\frac{E_s}{T_s}}\sum_{n=0}^{N-1} r_{n,i} w(t - iT_s - \tau_{n,i})(\cos(2\pi f_c t + \theta_i + \phi_n) + N(t)) \quad (1)$$

where $E_s$ is the energy per symbol, $w(t)$ is a unit amplitude rectangular pulse time limited to the interval $[0,T_s]$, $T_s$ is the symbol duration, $f_c$ the carrier frequency and $\theta_i = \{(2m-1)\pi/4, m=1..4\}$ the carrier phase. The noise term $N(t)$ is assumed to be an additive, zero-mean, white gaussian noise (*AWGN*) statistically independant of the multipath with spectral density $N_0/2$. The receiver is the classic coherent quarternary phase-shift keying receiver [5].

## III. SYMBOL ERROR RATE PERFORMANCE

The evaluation of the probability of error per symbol requires the knowledge of the probability density of the two decision variables X and Y, the output of the receiver in-phase and quadrature sub-branches. It is assumed, as in many previous research works, that the multipath components are represented by gaussian and mutually independant random variables. For instance, Geraniotis and Pursley in their analysis of performance of nonocoherent direct-sequence spread spectrum communications over specular multipath fading channels [6] represent by gaussian random variables the output branches of the DPSK modulation receiver for their DS/SS system . Bassel [4] represents the multipath components by zero mean gaussian random variables in his evaluation of the error probability of spread spectrum systems with BPSK modulation. For the same type of systems, Wang [1] has studied the effect of the power delay profile and represented the effect of intersymbol interferences by zero mean gaussian random variables.

This approach will aso be used in this work to analyze the symbol error rate performance of QPSK systems. Both Rayleigh and Rician selective fading channels are considered by using unilateral Maxwell and exponential delay profiles respectively. We assume that the number of echoes follows a Poisson distribution and that their arrival time is uniformly distributed. Using those assumptions, it can shown that the in-phase and the quadrature components X and Y outputs due to the interfering echos are uncorrelated with zero-mean and with the same variance: $\sigma_X^2 = \sigma_Y^2$. We introduce now the Fading Factor defined as: $F = 2\sigma_X^2 / E_b$.

### A. Power delay profiles

In this paper the performance of *QPSK* is considered for two examples of power delay profile. The Maxwell profile is given by:

$$p(\tau) = \alpha\tau^2 \exp(-\beta\tau^2)u(\tau) \qquad \alpha, \beta > 0 \tag{2}$$

while the exponential profile is given by:

$$p(\tau) = \alpha \exp(-\beta\tau)u(\tau) \qquad \alpha, \beta > 0 \tag{3}$$

where $\tau$ is the relative time delay and where the power delay profile is defined by:

$$p(\tau) = E\{r^2(\tau)/\tau\} \tag{4}$$

This expression physically means the power average of the echos arriving with time delay $\tau$.

## B. Exponential power delay profile

In the case of the exponential power delay profile model an *LOS* signal is present. We assume that the average power of the first path is stronger than that of the others paths, and that the receiver is synchronized to the first path signal; consequently, we can use $\tau_n = 0$ as time reference. The strongest interference in the delay signal comes right after the desired signal. Each delayed path arrives at the receiver with a relative delay time $\tau_n$ and amplitude $r_n$ and this amplitude will decrease following an exponential probability density [1,8]. We also assume that the intersymbol interference affects only adjacent data pulses, this condition being satisfied if the power delay profile $p(\tau)=0$ for $\tau > T_S$ [2]. Under this assumption the statistics of $r(t)$ depend on two consecutive data symbols $[S_i, S_{i+1}]$. A received symbol is then affected by the first part of its own interferenece and by the second part of the interference generated by the previous symbol. The exponential power profile of equation (3) is used to determine the variances $\sigma_X^2$ *a*nd $\sigma_Y^2$ :

$$\sigma_E^2 = F_E(E_b/2)$$
$$F_E = \frac{2\nu}{\xi^3}\left[(\xi-1)^2 + 3 - e^{-\xi}\left\{(\xi+1)^2 + 3\right\}\right] \tag{5}$$

where $\nu = E\{N\}$ is the average number of the echos arriving in the detection interval $[0,T_S]$ [3], $D$ is the root-mean-squared ( rms) delay spread and $\xi = T_S/D$.

## C. Maxwell power profile

In the Rayleigh fading channel, the received signal consists of the scattering components and of the additive channel noise. We assume that the receiver is synchronized to the maximum of the incoming signal, which occurs at $t_m = D\sqrt{2/(3-8/\pi)}$ . Equation (2) can be expressed as:

$$p(\tau) = \begin{cases} e\left(\frac{\tau + t_m}{t_m}\right)^2 \exp-\left(\frac{\tau + t_m}{t_m}\right)^2 & \tau \ge t_m \\ 0 & \tau < t_m \end{cases} \tag{6}$$

Under the assumption of adjacent-symbol limited intersymbol interference, the statistics of the receiver signal depend on three consecutive symbols represented by: $(S_{i-1}, S_i, S_{i+1})$. The variance $\sigma_M^2$ is determined by using the Maxwell power profile in eq. 6:

$$\sigma_M^2 = F_M(E_b/2)$$
$$F_M = \frac{2\nu e}{\xi^3}\int_0^{\xi}\chi^2[(\chi-1)^2+(\xi-|\chi-1|)^2]\exp[-\chi^2]d\chi \tag{7}$$

where $\xi = T_s/t_m$.

The symbol error rate performance of the system can be evaluated by assuming that the total effect of all significant echos can be represented by gaussian process:

$$P_e \approx erfc\sqrt{\frac{\gamma}{1+F\gamma}} \tag{8}$$

where $\gamma$ is the signal-to-noise ratio.

## IV. NUMERICALS RESULTS

In this section, the numerical results for the Symbol Error Rate (*SER*) with coherent *QPSK* modulation in Rayleigh and Rician selective fading channels are presented. The fading factor and the irreducible error probability are also discussed.

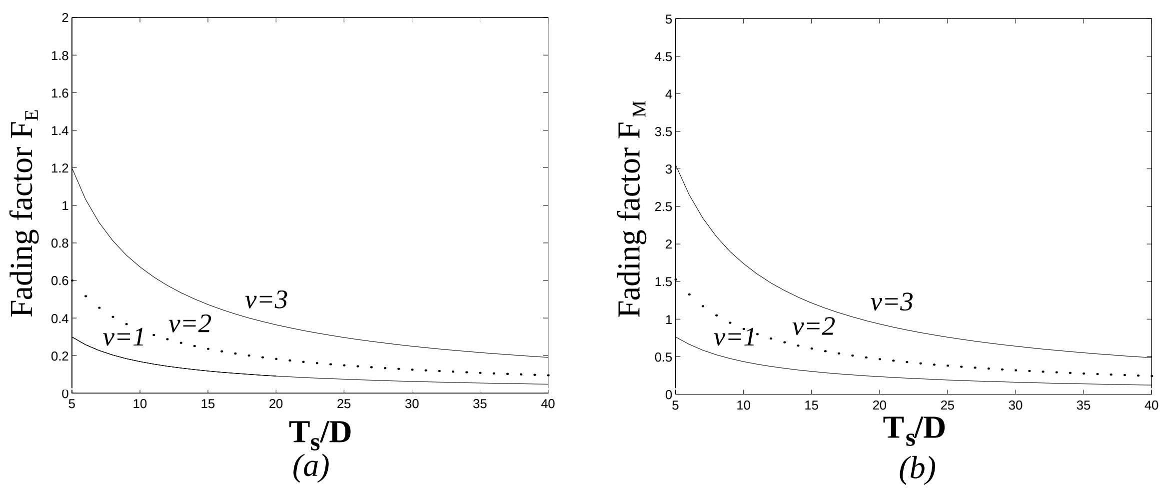


*Figure. 1 Fading factor for QPSK for (a) Exponential and (b) Maxwell delay power profiles for different number of scattering paths (ν)*

Fig.1.a and Fig.1.b show the fading factor $F$ for different values of the Poisson parameter $\nu$ as fonction of the normalized parameter $T_s$/D for Rician and Rayleigh fading channels. We can see that the fading factor decreases following a general exponential trend for small values of $T_s/D$

($T_S/D \leq 10$) and continues to decrease hyperbolically as $T_S/D$ increases. The number of scattering paths $\nu = E\{N\}$ changes its value with different types of areas [7,8] and has a strong influence on *F*.

In Fig. 2, the result of the numerical evaluation of the average error probability of *QPSK* modulation is illustrated as a fonction of the signal-to-noise ratio for different values of the fading factor. For the specific case where the fading factor is equal to zero, the performance reduces to the classical result for *QPSK* modulation in a non-selective fading channel. As the signal-to-noise ratio becomes larger, the effect of intersymbol interference becomes more important; at high signal-to-noise ratios the effect of the gaussian noise is negligible, and the error probabibility is essentially given by the irreducible error rate $P_I = erfc\sqrt{1/F}$ which depends on the fading factor. In this case we can say that $P_I$ is a good estimate for $P_e$.

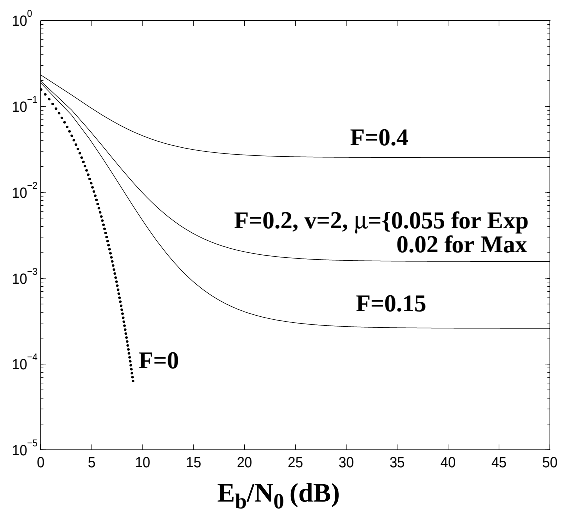


*Figure 2. SER in selective fading channels for several values of F compared to the probability of error of the classical QPSK modulation*

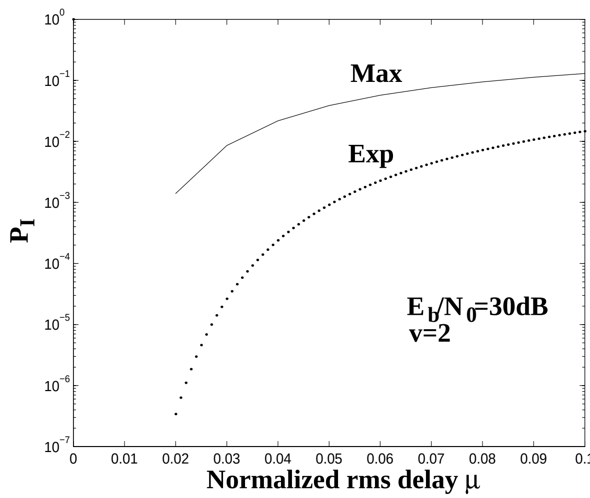


*Figure 3. The irreducible symbol error probability in Rayleigh and Rician channels as fonction of* μ

Figure 3 shows the irreducible error probability per symbol $P_I$ for the two power delay profile models in function of the normalized rms delay $\mu = T_S/D$. For pratical purposes the irreductible error probability is taken here as the value of the probability of error when the $E_b/N_0$ ratio equals 30dB. One notes that, for the same number of interfering signals, the presence of the line-of-sight component improves the system performance. The results of Figure 3 also indicate that, for the same ν, the irreducible error probability strongly depends on the type of power delay profile model, and is much lower for the exponential than for the Maxwell model.

## V. CONCLUSION

In this study, the performance of coherent *QPSK* communication system has been analysed. Expressions for the symbol error rate are derived in the context of selective Rayleigh and

Rician selective channels by using Maxwell and Exponential time delay spread models. The effects of the fading factor, the number of the scattering paths and the normalized rms delay $\mu$ on irreducible error probability have been developed. Compared to the case of additive white Gaussian noise, it is clear that the effect of the delay spread on the performance is very severe. Our results are similar to those of [2 ] where the Method of Moments is used for computing the error probability in the same conditions, namely same delay profile model and delay spread values.

The results of this study are particulary applicable to a broadband wireless personnal environment. The exponential and Maxwell delay profiles used here as examples are good representations of the power profiles met in many environments.

It is clear that, in order for QPSK to be useful for high data rate wide bandwidth systems in multipath fading channels, it is necessary to consider using diversity techniques, equalization, adaptive antennas. The approach for performance analysis presented here could be used as a first step in the design process of such more complex systems.

Comparisons of our results with those of other authors permit to conclude empirically that the gaussian approximation approach can be considered as useful for the performance analysis of systems operating in a multipath environment. In order to consolidate this approach, it would be of great interest to do similar comparative analyses with much more sophisticated methods, whose implementation is also much more arduous.

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