# Formation of A Supergain Array and Its Application in Radar

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*Abstract-* The formation of a linear array is introduced in [1]. When all of the element fields are in phase, we obtained a main beam with its maximum directivity. The phenomenon in which a linear array be able to have a gain which is higher than the maximum directivity of the normal linear array called superdirectivity [2,3,4]. In this paper, we study the formation of a supergain array and its application in Radar, some simulations are given to illustrate the array performance and some remark conclusions are given.

#### I. INTRODUCTION

In Radar modern, we interested in design a system having a very narrow beam and very high directivity. In general, in order to do so we have use a large array which is inconvenient in electronic-battle now a day. Therefore, the researches about super gain array have much attractive from Radar was born up to now. In this paper, the formation of a supergain array and its applications in Radar are presented. The paper is organized as follows. The directivity of a linear array is introduced in part II. Part III presents supergain array. Part IV presents simulation results. Part V discusses the application of the supergain array in Radar. The conclusions are given in part VI.

#### II. DIRECTIVITY OF A LINEAR ARRAY

Consider the case of a uniformly spaced linear array laid out along the z axis. We assume that the currents have equal amplitudes but a uniform progressive phase, i.e.

$$I_n = I_0 \exp(-jn\alpha) \tag{1}$$

With  $\alpha$  is a constant called the phase-shift factor. Under this assumption, the array factor is given by

$$A(\theta) = \sum_{-N}^{N} \exp[jn(kd\cos\theta - \alpha)]$$
(2)

If the element pattern is isotropic, the directivity is governed entirely by the array factor. It is defined as the power in the direction of the main beam maximum divided by the average power density from the array. Thus

$$D = \frac{A(\theta_0)A^*(\theta_0)}{\frac{1}{4\pi r_0^2} \int_0^{\pi} \int_0^{2\pi} A(\theta_0)A^*(\theta_0)r_0^2\sin\theta d\theta d\phi}$$
(3)

Where  $r_0$  is the distance from the antenna to the observation point,  $\theta$  is the angle from the boresight to the observation point,  $\theta_0$  is the angle from the boresignt to the main beam maximum.

Making use of (2) and d equals  $\lambda/2$ , this becomes

$$D = \frac{(\sum_{n=1}^{N} I_n)^2}{\sum_{n=1}^{N} I_n^2}$$
(4)

which is a most interesting formula in several respects. The directivity as given by (4), turn out to be a measure of the coherent of radiation from the linear array. The numerator is proportional to the total coherent field, squared, whereas the denominator is proportional to the sums of the squares of the individual fields from each element.

Furthermore, the directivity as given by (4) is seen to be independent of scan angle. On the face of it, this seems surprising, since we have already observed that the main beam broadens as it is scanned away from broadside, a manifestation which usually signifies lowered directivity. However, for a linear array, as the conical beam is scanned toward endfire, the core tends to occupy a smaller solid angle in space, an effect which just cancels the beam broadening. This compensation holds until the beam approaches endfire, when another compensation takes over- the appearance of a second main beam at reverse endfire.

Whereas, (4) is independent of scan angle, it is not independent of current distribution. The excitation can be expressed using the Fourier series description

$$I_{n} = \sum_{p=-P}^{P} a_{p} \exp(jp \frac{2\pi n}{2N+1})$$
(5)

Where *P* is the highest spatial harmonic needed to represent the distribution,  $a_p = a_{-p}$  is pure real because the distribution is assumed to be symmetric. Therefore, one find that

$$\left(\sum_{n=N}^{N} I_n\right)^2 = (2N+1)^2 a_0^2 \tag{6}$$
$$\sum_{n=N}^{N} I_n^2 = (2N+1) \sum_{p=N}^{P} a_p^2$$

Thus, (4) becomes

$$D = \frac{2N+1}{\sum_{-P}^{P} (a_p / a_0)^2}$$
(7)

For half-wave spacing,  $L = (2N+1)(\lambda/2)$  and uniform distribution, (7) becomes  $D = 2L/\lambda$  (8)

#### III. SUPERGAIN ARRAY

From (8), it is clearly that if we do not make any special changes in inter-element space or the phase excitation, the maximum directivity of the normal linear array being limited by the number of element used.

Define  $\psi$  as

$$\psi = \frac{1}{2} (kd\cos\theta - \alpha) \tag{9}$$

Assuming an endfire array with d is equal to half wave length, the phase shift factor is equal to kd, all element are isotropic radiation. The array factor is given by

$$A(\theta) = \frac{\sin(N\psi)}{N\sin\psi} \tag{10}$$

Hansen and Woodward in [2] proposed to scan the beam further than endfire, this causes an increase in the sidelobe level, but makes the "visible" portion of the main beam have a steeper average slope, giving rise to an increased directivity. The maximum directivity resulted when approximately half the main beam was scanned out of the visible region. In another way, the phase progressive in the constrain is given by

$$\alpha_s \approx \pm (kd + \frac{\pi}{N}) \tag{11}$$

The directivity of the supergain array is approximately given by

$$D_s = 7.28L/\lambda \tag{12}$$

## IV. SIMULATION RESULTS

Using Matlab program, we obtained the array factors of the endfire array with N varying from 2 to 6 as shown in Figure 4 to 6. To comparison purpose, we simulate the array factors of the supergain array with  $d = 0.3\lambda$  and N correspondingly as in shown in Figure 1 to 3.

From the Figure 4 to 6, we can see that the more element are used the narrower beam width obtained. When comparing three couples of figure, i.e. (1,4), (2,5), (3,6), the main beam width of the supergain array is always narrower than the one of the normal endfire array. In Figure 1 to 3, it is clearly that the maximum of the main beam of the supergain array is out of the visible region. This caused an increasing of transmitted energy in the main beam compared to the average transmitted energy in visible region. Therefore, there is an enhanced directivity.



## V. APPLICATIONS TO RADAR

A Radar's antenna system should have very high directivity, so we suggest to use two supergain arrays with two elements ( $D_s = 2.184$ ) combining with signal processing.

As in [4], in principle, to obtain the super directional pattern we have to realize the directional pattern with the phase jump by  $\pi$  in any direction, while the amplitude in that direction remains the same.

A typical phase characteristic needed for such purpose is shown in the Figure 7 as follows



Figure 7. Phase pattern of the first and second array

When using two arrays, the phase patterns of which are axial-symmetrical to each other, we readily to see that within the interval of direction  $\alpha_M \pm \delta$ , with  $\delta \rightarrow 0$ , we transmit the maximum voltage, while in the other direction we transmit nothing. It is evident because in direction  $\alpha_M - \delta \le \alpha \le \alpha_M + \alpha$  the transmitted signal from both array are quite in phase, but in the other direction the signal are quite anti-phase to each other. Finally, the Radar's antenna system has superdirectivity not only by supergain array but also by using axial-symmetrical phase scheme.

# VI. CONCLUSIONS

A Radar's antenna system is introduced using two supergain arrays with two elements combining with the axial-symmetrical phase scheme. Its high performance is very attractive but the realization of the system should be investigated.

# REFERENCES

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