An Approach for Passive Radar Using
A Smart Antenna System

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Abstract- An antenna without phase center system used for DOA estimation and beam steering was introduced in [1], [2]. In the radar problem, the antenna can receive the signals scattered from a moving target. In this paper, an approach for passive radar using a smart antenna system is introduced. We assume that the primary transmitted signal comes from a known fix voice broadcasting station. The principle of the passive radar are presented.

Keyword: Passive Radar, antenna without phase center, Doppler frequency, Direction of arrival.

I. INTRODUCTION

An active radar using a pulse sequence was introduced in [3]. In this case, we can use some kind of transmitted signal such as a coded pulse sequence or a modulated analog waveform, etc. One shortcoming of the active radar is that the enemy can find its location by its transmitted sequence. Therefore, a passive radar becomes a high priority in term of its safety. In addition, when the smart antenna is introduced, the capability of combining both antenna and passive radar make the radar system more efficient.

This paper is organized as follows. The active radar principle is presented in section II. The smart antenna structure is described in section III. The operation of the passive radar using the smart antenna system is discussed in section IV. Section V concludes the paper.

II. ACTIVE RADAR PRINCIPLE

As in [3], in the detection problem, an active radar transmits a cosine wave continuously. Thus

\[ s_i(t) = \sqrt{2} \text{Re}[\sqrt{E_i} e^{j\omega_d t}] \]

where \(E_i\) is the transmitted energy.

The returning wave is given by

\[ s_i(t) = \sqrt{2} \text{Re}[\sqrt{E_i} \tilde{b} f(t-\tau) e^{j\theta_i}] \] (2)

where \(\tilde{b}\) is a complex Gaussian random variable,

\(\tau\) is the round trip delay and \(f(t)\) is the complex envelope of the transmitted signal.

The total received waveform in white bandpass noise is

\[ r(t) = \tilde{b} \sqrt{E_i} f(t-\tau) e^{j\omega_d t} + n(t) \] (3)

where \(\omega_d\) is the Doppler frequency caused by relative movement between the target and the radar and \(n(t)\) is the white complex Gaussian random process.

In the binary hypothesis testing, we just take the correlation between the complex received waveform and the complex envelope of the transmitted signal. Then we do the square-law envelope detection and the threshold comparison to make the decision whether the target is present or not.

For DOA estimation problem, when a target moving, we have to consider the generalized ambiguity function, the generalized correlation function [3]. Derivating the generalized log likelihood function in azimuthal angle and equating it to zero, we obtain the desired estimation value. The disadvantages of this method are its complexity construction of the generalized ambiguity function and its heaviness of computation.

III. SMART ANTENNA STRUCTURE

The selected smart antenna system is constructed on the base of an antenna without phase center [4] (Figure 1). It consists of two elements. The first element is a vertical monopole with an omni-directional phase pattern, i.e.

\[ \Phi_1(\theta) = C \] (4)
where $\theta$ is the angular coordinate of the observation point. The second element is without phase center which is a combination of two couples of vertical dipoles. All vertical dipoles have equal amplitude excitation, $I_0$. The Cartesian coordinates of the dipoles and the phase of their excitation currents are as follows:

- Number 1 $(0, -d_1/2, 0)$; $\alpha_1 = 0$ radian
- Number 2 $(0, d_1/2, 0)$; $\alpha_2 = \pi$ radian
- Number 3 $(d_2/2, 0, 0)$; $\alpha_3 = \pi / 2$ radian
- Number 4 $(-d_2/2, 0, 0)$; $\alpha_4 = 3\pi / 2$ radian

The phase pattern of the second element can be written as

$$\Phi_2(\theta) = \arctan \left[ \frac{kd_1 \sin \theta}{kd_2 \cos \theta} \right]$$  \hspace{1cm} (5)

In general, $\Phi_2(\theta)$ is a nonlinear function of $\theta$.

If we choose $kd_1$ and $kd_2 \ll 1$, then we have

$$\Phi_2(\theta) = \theta$$  \hspace{1cm} (6)

We use the smart antenna system as the receiving antenna for our passive radar system.

### IV. Passive Radar System

We assume that a fix voice broadcasting station transmits an AM signal as

$$x_c(t) = G_0 (1 + \mu x(t)) \cos \omega t$$  \hspace{1cm} (7)

Where $G_0(1 + \mu x(t))$ is the envelope of the AM transmitted signal, denoted by $A(t)$, $\omega$ is the carrier frequency, $\mu$ is the modulation index ($0 < \mu < 1$) and $x(t) = \cos \Omega t$ is the message signal.

The reflected signal from the target is given by

$$y_c(t) = \sqrt{2} \text{Re}[b_1 A(t - \tau_1) e^{j\omega \tau_1}]$$  \hspace{1cm} (8)

where $b_1$ is the complex Gaussian random variable ($E(b_1) = 0$, $E(b_1^2) = \sigma^2$) and $\tau_1$ is the delay of the transmitted signal.

The received signal at the front end of the passive radar in white bandpass noise is given by

$$r_c(t) = y_c(t) + n(t)$$  \hspace{1cm} (9)

The noise is white Gaussian distribution with its spectral density is equal to $N_0 / 2$.

In detection problem, because we use the smart antenna system as depicted in Figure 1, the signal going out of two couples of dipole $I=1+3$ and $II=2+4$ in the absent of noise are $r_1$ and $r_2$, respectively.

$$r_1 = \text{Re}[G_0 b_1 (1 + \mu \cos \Omega t) e^{j\omega t}]$$  \hspace{1cm} (10)

$$r_2 = \text{Re}[jG_0 b_1 (1 + \mu \cos \Omega t) e^{j\omega t}]$$

where $G_0 = I_0 l$ is the amplitude pattern with $l$ being the length of a dipole.

We rewrite (10) in the form

$$r_1 = G_0 \bar{b}_1 \cos \omega t + G_0 \bar{b}_1 \frac{\mu}{2} \cos(\omega - \Omega) t$$

$$+ G_0 \bar{b}_1 \frac{\mu}{2} \cos(\omega + \Omega) t$$  \hspace{1cm} (11)

$$r_2 = G_0 \bar{b}_1 \sin \omega t + G_0 \bar{b}_1 \frac{\mu}{2} \sin(\omega - \Omega) t$$

$$+ G_0 \bar{b}_1 \frac{\mu}{2} \sin(\omega + \Omega) t$$
It can easily be seen that the pair of two couples of dipole in this case produce three pairs of the component currents. These currents of each pair are in quadrature. If we denote

\[ s_{11}(t) = G_0 b_1 \cos \omega t, \quad s_{21}(t) = G_0 b_1 \sin \omega t \]
\[ s_{12}(t) = G_0 b_1 \frac{\mu}{2} \cos(\omega - \Omega) t \]
\[ s_{22}(t) = G_0 b_1 \frac{\mu}{2} \sin(\omega - \Omega) t \]
\[ s_{13}(t) = G_0 b_1 \frac{\mu}{2} \cos(\omega + \Omega) t \]
\[ s_{23}(t) = G_0 b_1 \frac{\mu}{2} \sin(\omega + \Omega) t \]

Then we have

\[ L_{1i} = \int_0^T r_i(t)s_{1i}(t)dt, \quad i = 1, 2, 3 \]
\[ L_{2i} = \int_0^T r_i(t)s_{2i}(t)dt, \quad i = 1, 2, 3 \]

where \( T \) is the observation period and (11) can be rewritten as

\[ r_1 = \sum_{i=1}^{3} s_{1i}, \quad r_2 = \sum_{i=1}^{3} s_{2i} \]

The sufficient statistic is given by

\[ l = \frac{\sigma^2}{\sigma^2 + N_0} \sum_{i=1}^{3} (L_{1i}^2 + L_{2i}^2) \]

In order to make the decision whether the target present or not, we compare the sufficient statistic with the threshold. Therefore, the optimum receiver A is depicted as in Figure 2.

Then we construct the optimum receiver B (Figure 3) to estimate the DOA. The AM signal with frequency \( \omega \) is received by the first element with the phase pattern (4) and being envelope detected. Thus we have the output signal is given by

\[ r_{1B} = G_0 (1 + \mu \cos \Omega t) \]

It is straightforward from (7).

The reflected signal from the target with frequency \( \omega + \omega_d \) is received by the second element with the phase pattern (6) and being envelope detected. Thus we have the output signal is given by

\[ r_{2B} = G_0 \tilde{b}_2 \mu \cos(\theta + \Omega t) \]

where \( \tilde{b}_2 \) and \( \tilde{b}_1 \) are identical distribution.

The first pair of the component currents in (11) expresses the carrier field without phase modulation. The second and the third pair of the component currents in (11) express the envelope field with their phases are symmetrical if we change the side of \( \sin(\omega - \Omega) t \) from positive to negative. In addition, the output of the second element has its phase pattern is proportional to the DOA so we have (17). Finally, \( r_{1B} \) and \( r_{2B} \) being phase compared, the phase different between them is the desired DOA.

V. CONCLUSIONS

In this paper, we introduce the application of the smart antenna system for a passive radar. The transmitted signal from known fix broadcasting stations is exploited for our purpose. Because these stations already exist, their coverages are nation wide and the optimum receivers are simple, the system is very useful and this research can be used for other researches in radar technique.

REFERENCES

\[
\frac{\sigma^2}{\sigma^2 + N_\omega} \cos \omega t \\
\frac{\sigma^2}{\sigma^2 + N_0} \cos (\omega + \Omega) t \\
\frac{\sigma^2}{\sigma^2 + N_\omega} \cos (\omega - \Omega) t \\
\frac{\sigma^2}{\sigma^2 + N_0} \sin \omega t \\
\frac{\sigma^2}{\sigma^2 + N_0} \sin (\omega + \Omega) t \\
\frac{\sigma^2}{\sigma^2 + N_\omega} \sin (\omega - \Omega) t
\]

Figure 2. Optimum receiver A