Combining belief functions and fuzzy membership functions

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ABSTRACT

In several practical applications of data fusion and more precisely in object identification problems, we need to combine imperfect information coming from different sources (sensors, humans, etc.), the resulting uncertainty being naturally of different kinds. In particular, one information could naturally been expressed by a membership function while the other could best be represented by a belief function. Usually, information modeled in the fuzzy sets formalism (by a membership function) concerns attributes like speed, length, or Radar Cross Section whose domains of definition are continuous. However, the object identification problem refers to a discrete and finite framework (the number of objects in the data base is finite and known). This implies thus a natural but unavoidable change of domain. To be able to respect the intrinsic characteristic of uncertainty arising from the different sources and fuse it in order to identify an object among a list of possible ones in the data base, we need (1) to use a unified framework where both fuzzy sets and belief functions can be expressed, (2) to respect the natural discretization of the membership function through the change of domain (from attribute domain to frame of discernment). In this paper, we propose to represent both fuzzy sets and belief function by random sets. While the link between belief functions and random sets is direct, transforming fuzzy sets into random sets involves the use of α-cuts for the construction of the focal elements. This transformation usually generates a large number of focal elements often unmanageable in a fusion process. We propose a way to reduce the number of focal elements based on some parameters like the desired number of focal elements, the acceptable distance from the approximated random set to the original discrete one, or the acceptable loss of information.

Keywords: Random sets, fuzzy sets, belief functions, object identification, information fusion

1. INTRODUCTION

For twenty years, uncertainty has been considered as an inherent part of fusion problems since information is never perfect, nor precise, nor certain. Although uncertainty was considered first as a synonym of probability, it appeared that many other aspects (kinds) of uncertainty cannot be captured by the concept of probability, and thus this enhances some lacks of theory of probability. To overcome these lacks, new theories are born such as fuzzy set theory, evidence theory, possibility theory, random set theory, rough set theory, etc. Even if each of these theory can be applied to any problem involving uncertainty, none of them is better than another. Instead they address different kinds of uncertainty thus they are not competitors but they rather complete each other. For example, probability theory well suit to random data, where a probability distribution can be established, whereas fuzzy set theory concerns vague or fuzzy data, *i.e.* where boundaries are not well defined.

In data fusion problems, we need to combine information coming from multiple and variate sources. Even if the variety of the sources is probably a great asset, it can however induce another problem that is the choice of the "best" theory to model the information. Depending of the features of the information source, the uncertainty can be of different kinds and thus many different theories could be used. In order to take into account this problem, some unifications among theories of uncertain reasoning are currently under study. A good candidate

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seems to be random set theory whose framework can represent most of the theories. A preliminary study on this ability was this objective of a previous paper.[1]

In this paper, we consider the problem of target (object) identification using multiple sources of information. Typically, these sources can be (1) measures devices such as radars (long and short range), ESM (Electronic Support Measures), IFF (Interrogator Friend or Foe), but also (2) human sources such as opinions of expert observing the scene. The first category of sources is well modeled by belief functions, however fuzzy set theory better corresponds to the second category since it is naturally designed to model human language descriptions. For example, an expert can affirm "*the target is small*" or "*the target has a quite low speed*", information specially well represented by membership functions. Hence, here we propose to put in the random set framework both belief functions and fuzzy membership functions in order to combine them for identifying the observed target.

Section 2 is dedicated to some theoretical basics first on random sets (section 2.1) and then successively to evidence theory (section 2.2) and fuzzy sets (section 2.3). In each subsection, the representation of the theory in the random set framework is given, and the modelization of information is described. In particular, the change of definition domain for membership functions is mentioned in section 2.3.1. We approach the problem of the approximation of the random set issued from the membership transformation in section 3. Indeed, the number of focal elements involved by such a transformation is often unmanageable for a fusion process, hence a reduction of this number is unavoidable. Finally, the mechanism of combination of belief functions and fuzzy sets in the random set formalism is then described in section 4 through an example of application to target identification, with three kinds of reports: ESM reports, Radar reports and expert opinions. Section 5 is the conclusion.

2. THEORETICAL BASICS

2.1. Random set theory

The random set concept has been introduced in the early 70's[2,3] as a generalization of the random variable concept: Random sets are random elements whose values are sets, whereas random variables are random elements whose values are numbers. Hence, roughly speaking, random set theory may be viewed as a generalization of random variables and vectors.

Let $(\Omega, \mathbf{A}, P)$ be a finite and discrete probability space and let Θ be a finite discrete set. A **random set** $\mathcal{X}$ is defined by a (multivalued) mapping $\mathcal{X} : \Omega \to 2^\Theta$ where 2^Θ is the power set of Θ. Any probability measure defined by a probability distribution function $f : 2^\Theta \to [0, 1]$ such that

$$f(A) = P[\mathcal{X} = A] \qquad \forall A \subseteq \Theta \tag{1}$$

defines a random set $\mathcal{X}$ on Θ. $[\mathcal{X} = A]$ is the event that a randomly selected subset of Θ be equal to the particular subset A.

This probability distribution f entirely characterizes the random set. Also, it satisfies the same axioms as every probability distribution:

$$P[\mathcal{X} = \emptyset] = 0 \tag{2}$$

$$0 \leq P[\mathcal{X} = A] \leq 1 \quad , \forall A \subseteq \Theta \tag{3}$$

$$\sum_{A \subseteq \Theta} P[\mathcal{X} = A] = 1 \tag{4}$$

From f we can define three other useful functions entirely characterizing the random set:

1. **the implying functional** $R_\mathcal{X}$:

$$R_\mathcal{X}(A) = P[A \subseteq \mathcal{X}] \qquad , \forall A \subseteq \Theta; \tag{5}$$

2. **the hitting capacity** $T_\mathcal{X}$:

$$T_\mathcal{X}(A) = P[\mathcal{X} \cap A \neq \emptyset] \qquad , \forall A \subseteq \Theta; \tag{6}$$

3. **the inclusion capacity** $P_{\mathcal{X}}$:

$$P_{\mathcal{X}}(A) = P[\mathcal{X} \subseteq A] \qquad , \forall A \subseteq \Theta. \tag{7}$$

A simplified view of a random set $\mathcal{X}$ allows to represent it by a set of couples:

$$\mathcal{X} = \{(A_i, m_i) | A_i \subseteq \Theta, m_i = P[\mathcal{X} = A_i], 1 \leq i \leq 2^N, \sum_{i=1}^{2^N} m_i = 1\}, \tag{8}$$

where $N = \text{card}(\Theta)$.

Recently, Goodman, Malher and Ngyuen[4,5] among other authors presented random set theory as a unifying paradigm for most of theories of uncertain reasoning. It appears that at least probability theory, Dempster-Shafer theory, possibility theory, fuzzy sets theory and conditional events algebra can be represented in the random set framework. Hence, such an unification offers a systematic methodology for the fusion of information involving various types of uncertainty. In this paper, we restrict our discourse to evidence theory and fuzzy sets theory.

2.2. Evidence theory

Evidence theory is a powerful tool to deal with imprecise and uncertain information, developed by Dempster[6] and later on by Shafer.[7] This theory is often described as an extension of probability theory as it lies on the power set of the universe of discourse instead of the set itself. Among other properties, the additivity axiom of probability theory is replaced by a super-additivity one.

Let Θ be the frame of discernment, containing N objects, hypotheses, etc. A Basic Probability Assignment (BPA) is a mapping defined as $m : 2^\Theta \to [0, 1]$ that must satisfy the following conditions:

$$m(\emptyset) = 0 \tag{9}$$

$$0 \leq m(A) \leq 1 \quad , \forall A \in 2^\Theta \tag{10}$$

$$\sum_{A \subseteq \Theta} m(A) = 1 \tag{11}$$

$m(A)$ represents the degree of belief (confidence) that someone assigns strictly to A. A subset A with a non-null mass is called a **focal element** (of m).

From a BPA m, we can define three other functions from 2^Θ to $[0, 1]$:

1. the **belief function** Bel:

$$\text{Bel}(A) = \sum_{B \subseteq A} m(B) \qquad , \forall A \subseteq \Theta. \tag{12}$$

   Bel(A) represents the total belief that someone could assign to the subset A.

2. the **plausibility function** Pl:

$$\text{Pl}(A) = \sum_{B \cap A \neq \emptyset} m(B) \qquad , \forall A \subseteq \Theta. \tag{13}$$

   Pl(A) represents the maximal belief someone could assign to A (the belief to which someone finds A credible).

3. the **commonality function**, Q:

$$Q(A) = \sum_{A \subseteq B} m(B) \qquad , \forall A \subseteq \Theta. \tag{14}$$

   If the mass $m(A)$ is considered to be the probability degree that can move freely to any subset of A, then the commonality degree $Q(A)$ is viewed to be the total degree of probability that can move freely to any subset of A.

Between the basic probability assignment, the belief function, the plausibility function and the commonality function there exist bijective transformations:

$$m(A) = \sum_{B \subseteq A} (-1)^{|A-B|} \mathrm{Bel}(B) \qquad , \forall A \subseteq \Theta \tag{15}$$

$$\mathrm{Bel}(A) = 1 - \mathrm{Pl}(\overline{A}) \qquad , \forall A \subseteq \Theta \tag{16}$$

$$\mathrm{Bel}(A) = \sum_{B \subseteq \overline{A}} (-1)^{|B|} Q(B) \qquad , \forall A \subseteq \Theta \tag{17}$$

where $|A| = \mathrm{card}(A)$ and $\overline{A}$ is the complement of A relatively to Θ.

Let m_1 and m_2 be two BPAs defined on the same frame of discernment Θ. Combining information from the two BPAs is frequently done using Dempster's rule of combination,[6] which is a conjunctive rule:

$$(m_1 \oplus m_2)(A) = \frac{\sum_{B \cap C = A} m_1(B) m_2(C)}{1 - \sum_{B \cap C = \emptyset} m_1(B) m_2(C)} \qquad , \forall A \subseteq \Theta. \tag{18}$$

The weight of conflict between two BPAs is defined by:

$$\mathrm{Con}(m_1, m_2) = -\log\Big(1 - \sum_{B \cap C = \emptyset} m_1(B) m_2(C)\Big). \tag{19}$$

and two BPAs are totally in conflict if $\mathrm{Con}(m_1, m_2) = +\infty$. In this case, Dempster's combination cannot be applied.

2.2.1. Representing information in the evidence theory

Let $\Theta = \{\theta_1, \theta_2, \ldots, \theta_N\}$ be a set of objects, and let each object having a set of known parameters. A sensor provides information such as "*the observed object is a ship with a degree of confidence of 0.8*". This kind of information can be modeled in the evidence theory by a BPA m, as for example *:

$$\begin{array}{ll} A_1 = \{\theta \in \Theta | \theta \text{ is a ship}\} & m(A_1) = m_1 = 0.8 \\ A_2 = \Theta & m(A_2) = m_2 = 0.2 \end{array}$$

2.2.2. Random set representation for the belief functions

It can be proved (Nguyen,[8] Quinio and Matsuyama[9]) that the hitting capacity, the implying functional and the inclusion capacity from the random set theory are equivalent respectively to the plausibility function, the belief function and the commonality function from the evidence theory, the probability distribution function f being thus equivalent to the BPA m:

$$f(A) = P[\mathcal{X} = A] = m(A) \tag{20}$$

$$R_{\mathcal{X}}(A) = P[A \subseteq \mathcal{X}] = \mathrm{Bel}(A) \tag{21}$$

$$T_{\mathcal{X}}(A) = P[\mathcal{X} \cap A \neq \emptyset] = \mathrm{Pl}(A) \tag{22}$$

$$P_{\mathcal{X}}(A) = P[\mathcal{X} \subseteq A] = Q(A) \tag{23}$$

Moreover, Dempster's rule of combination (18) is a particular case of the intersection of random sets (when the random sets are independent):

$$T_{\mathcal{X}_1 \oplus \mathcal{X}_2}(A) = P(\mathcal{X}_1 \cap \mathcal{X}_2 = A | \mathcal{X}_1 \cap \mathcal{X}_2 \neq \emptyset), \forall A \in 2^{\Theta}. \tag{24}$$

The evidence theory is a simplified view of the random set theory (Nguyen[8] and Nguyen and Wang[10]) and the notation (8) shows the relation between the two mathematical models.

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*Note that here m_1 and m_2 are the two values of the same BPA m and must not be confused with two distinct BPAs, as used in (18)

2.3. Fuzzy set theory

Fuzzy set theory was developed by Zadeh[11] to model vague information such human language descriptions ("*small, large, quick, young*"), concepts that cannot be defined by an interval with strict limits. A fuzzy set is thus a set whose boundaries are not precise, not well defined, *i.e. fuzzy*. It is a more general concept of the classical set: the membership of an element to a fuzzy set is not described by a Boolean function (as it is the case for a classical set), but by real values between 0 and 1, in general (note that it can also be any other function). The theory of fuzzy sets is thus a generalization of the classical theory of sets.

In classical set theory a subset A of the frame of discernment Θ is represented by a membership function:

$$\begin{aligned} &\mu_A : \Theta \to \{0,1\} \\ &\mu_A(\theta) = \begin{cases} 1 & \text{if} \quad \theta \in A \\ 0 & \text{elsewhere} \end{cases} \qquad \forall \theta \in \Theta. \end{aligned} \tag{25}$$

A is referred to as a crisp set or a classical set.

A fuzzy set, denoted by $\underline{A} \subseteq \Theta$, is defined by a membership function which can take its values in the $[0,1]$ interval:

$$\mu_{\underline{A}}(\theta) \in [0,1] \qquad \forall \theta \in \Theta. \tag{26}$$

2.3.1. Representing information using the fuzzy sets theory

Let $\Theta = \{\theta_1, \theta_2, \ldots, \theta_N\}$ be a set of objects, and let each object have a set of known parameters:

$$\theta_i = [x_1^i, x_2^i, \ldots, x_P^i]^T.$$

Let consider that the information concerning the parameter x_j is vague and let $\mathcal{D}_j$ be the definition domain of this parameter ($x_j \in \mathcal{D}_j$). $\mathcal{D}_j$ can be discrete or continuous, ordered or not. Let $\underline{A} \subseteq \mathcal{D}_j$ be the fuzzy set with a membership function $\mu_{\underline{A}}(x_j)$. The fuzzy set $\underline{A} \subseteq \mathcal{D}_j$ must be transformed into a fuzzy set of $\underline{B} \subseteq \Theta$ relatively to the parameter x_j. After this transformation, the resulting subset is no more ordered nor continuous according to the parameter x_j:

$$\mu_{\underline{B}}(\theta_i) = \mu_{\underline{A}}(x_j^i). \tag{27}$$

For example, the parameter x_j could correspond to the *length* of the objects from Θ and the fuzzy set $\underline{A} \subseteq \mathcal{D}_j$ could correspond to *small length*. In this case, $\mathcal{D}_j$ is continuous and ordered. Each value from the interval $[0,300]$ meters possesses a membership degree to the fuzzy subset *small length* (see left graphic of the figure 1). Using the values x_j^i of the parameter x_j for each object θ_i of Θ, we construct a new fuzzy set $\underline{B} \subseteq \Theta$ where $\mu_{\underline{B}}(\theta_i)$ represents the degree of membership of each object of Θ to the fuzzy set *objects with small length* (see right graphic of the figure 1).

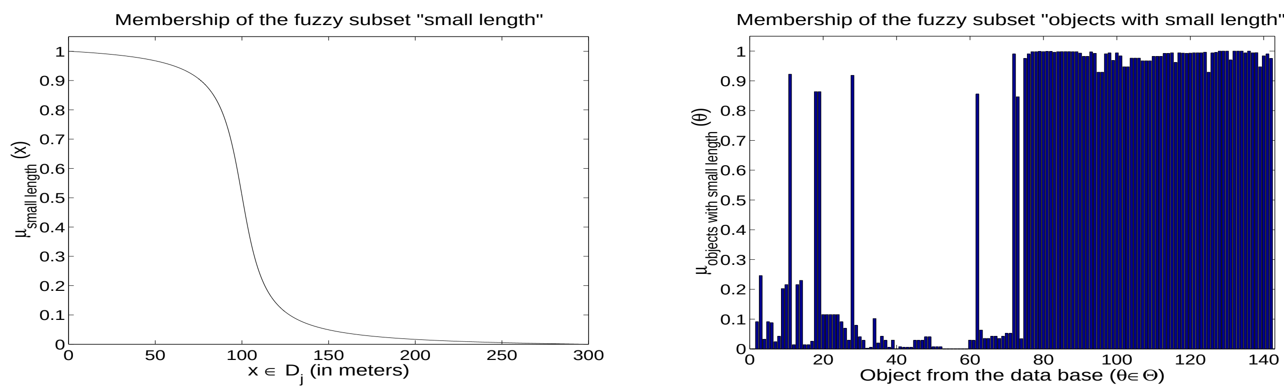

Figure 1: Membership of the fuzzy set "small length" and of the fuzzy set "objects with small length"

2.3.2. Random set representation for membership functions

In,[12] a formal connection between random sets and fuzzy sets is proposed, fuzzy sets being considered as equivalence classes of random sets, "one-point coverages" of random sets.

Let $(\Omega, \mathbf{A}, P)$ be a probability space and Θ be a finite space. Then, with each random set $\mathcal{X}$ from Ω to 2^Θ we can associate a membership function $\mu_\mathcal{X} : \Theta \longrightarrow [0,1]$ of a fuzzy set on Θ, such that

$$\mu_\mathcal{X}(\theta) = P\left(\theta \in \mathcal{X}\right), \forall\theta \in \Theta \tag{28}$$

$\mu_\mathcal{X}(\theta)$ is the *one-point covering function* of $\mathcal{X}$.

In practice, the α-cut representation of a fuzzy set is used to construct the corresponding random set. Let $\underline{A} \subseteq \Theta$ be a fuzzy set defined by the membership function $\mu_{\underline{A}}$. An α-cut of $\underline{A}$ is the subset A_α of Θ such that:

$$A_\alpha = \{\theta | \mu_{\underline{A}}(\theta) \geq \alpha\}. \tag{29}$$

Hence, $\underline{A}$ can be represented by a stack of α-cuts.

Suppose the fuzzy set is defined on a **discrete** domain and let $0 \leq \alpha_1 \leq \alpha_2 \leq \cdots \leq \alpha_M \leq 1$ be the distinct values of the membership function $\mu_{\underline{A}}$ ($M \leq \text{card}(\Theta)$). Then, the fuzzy set is defined in the random set theory using the α-cut representation:

$$A_i = \{\theta | \mu_{\underline{A}}(\theta) \geq \alpha_i\} \quad \forall i\ ,\ 1 \leq i \leq M \tag{30}$$

$$m_i = \begin{cases} \dfrac{\alpha_i - \alpha_{i-1}}{\alpha_M} & \forall i\ ,\ 2 \leq i \leq M \\ \dfrac{\alpha_1}{\alpha_M} & i = 1 \end{cases} \tag{31}$$

This representation exactly defines $\underline{A}$.

However, M is often too large and we need to restrict the number of focal elements to only $M_0 \ll M$ so that Dempster's rule of combination can be applied without algorithm explosion (*i.e.* using a reasonable computation time).

3. APPROXIMATION OF MEMBERSHIP FUNCTIONS

The fusion process using Dempster's rule is often computationally highly expensive because the transformations of the fuzzy memberships into the random set model involve too many focal elements. Indeed, let consider the transformation based on the α-cuts described by equations (30) and (31). Let call M_0 the desired number of focal elements of the random set built from $\mu_{\underline{A}}$, $M_0 \ll M$. To reduce the number of focal elements, we must find a new set of α-cuts ($0 \leq \alpha_1 \leq \alpha_2 \leq \cdots \leq \alpha_{M_0} \leq 1$) differing from the original (and "optimal") one. The random set representation becomes then:

$$A_i = \{\theta | \mu_{\underline{A}}(\theta) \geq \alpha_i\} \quad \forall i\ ,\ 1 \leq i \leq M_0 \tag{32}$$

$$m_i = \begin{cases} \dfrac{a_i - a_{i-1}}{a_{M_0}} & \forall i\ ,\ 2 \leq i \leq M_0 \\ \dfrac{a_1}{a_{M_0}} & i = 1 \end{cases} \tag{33}$$

where

$$\begin{cases} a_{M_0} = \max\{\mu_{\underline{A}}(\theta) | \theta \in \Theta\} \\ a_i = \max\{\mu_{\underline{A}}(\theta) | \theta \in \Theta \setminus A_{i+1}\} \quad \forall i\ ,\ 1 \leq i \leq M_0 - 1 \end{cases} \tag{34}$$

When no approximation is made $M_0 = M$ and $a_i = \alpha_i, 1 \leq i \leq M$.

Hence, using (32) and (33) instead of (30) and (31) allows to avoid an algorithm explosion. However, this approximation can depend on different parameters such as the α-step (difference between two α-levels) or more generally the level set (*i.e.* the set of α-cuts), or the desired maximum number of focal elements, M_0. In this work, we simply consider constant α-steps (we will denote by α), and more general level sets will be the aim of future studies. In this particular case, the constant α is related to the maximum number of focal elements of the approximated representation by:

$$M_0 = \left\lceil \frac{1}{\alpha} \right\rceil \tag{35}$$

To compare differences between the random sets obtained without any approximation and those obtained using an α-cut approximation we propose to use a distance between the two random sets, especially that proposed by Jousselme and al. in[13, 14] for the Dempster-Shafer theory that can easily be transposed to the random set theory. Let $\mathcal{X}_1$ and $\mathcal{X}_2$ be two random sets defined on Θ, then $d_{RS}(\mathcal{X}_1, \mathcal{X}_2)$ quantifies the distance between them (in the 2^Θ space) with:

$$d_{RS}(\mathcal{X}_1, \mathcal{X}_2) = \sqrt{\frac{1}{2} < \mathcal{X}_1 - \mathcal{X}_2, \mathcal{X}_1 - \mathcal{X}_2 >} = \sqrt{\frac{1}{2} < \mathcal{X}_1, \mathcal{X}_1 > - < \mathcal{X}_1, \mathcal{X}_2 > + \frac{1}{2} < \mathcal{X}_2, \mathcal{X}_2 >} \tag{36}$$

where:

$$< \mathcal{X}_1, \mathcal{X}_2 > = \sum_{A \subseteq \Theta} \sum_{B \subseteq \Theta} P[\mathcal{X}_1 = A] P[\mathcal{X}_2 = B] \frac{\text{card}(A \cap B)}{\text{card}(A \cup B)}. \tag{37}$$

In table 1 we introduce three vague pieces of information initially modeled by the fuzzy set theory. To simplify the fusion process, the number of focal elements must be reduced for each one of these information. Thus each fuzzy set must be approximated by a random set which is as close as possible (as desired) to the representation without approximation. Let $\mathcal{X}_t^0$ be the random set representation without approximation of the information at instant t, $t \in \{3, 4, 7\}$. Let $\mathcal{X}_t^\alpha$ be the random set representation approximated using this level set (for information of instant t).

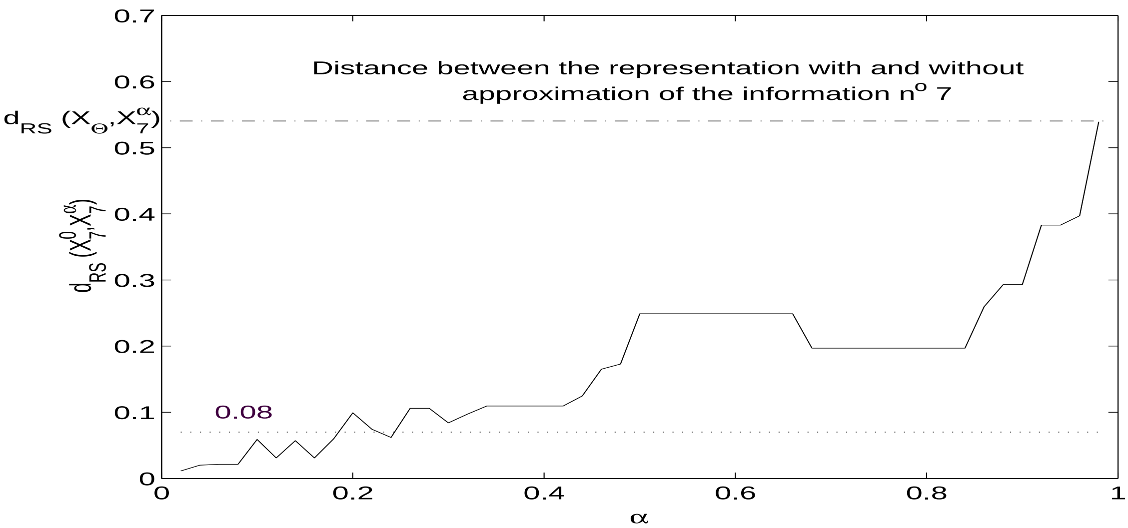

Figure 2: Distance between $\mathcal{X}_7^0$ and $\mathcal{X}_7^\alpha$

Figure 2 presents the distance between $\mathcal{X}_7^0$ and $\mathcal{X}_7^\alpha$. The case $\alpha = 1$ corresponds to the distance between $\mathcal{X}_7^0$ and the total ignorance ($\mathcal{X}_\Theta$, the random set with probability distribution such that $P[\mathcal{X}_\Theta = \Theta] = 1$). If we approximate a fuzzy set by $\mathcal{X}_\Theta$, this information is eliminated from the fusion process (total ignorance is the neutral element for Dempster's combination).

Hence, for any imposed threshold on the distance $d_{RS}(\mathcal{X}_7^0, \mathcal{X}_7^\alpha)$ we can find a maximum value of α for which the approximation is never farer from the optimal random set $\mathcal{X}_7^0$ than this threshold. In our example, we chose a threshold equal to 15% of $d_{RS}(\mathcal{X}_\Theta, \mathcal{X}_7^\alpha)$, the distance corresponding to the worse approximation. On figure 2, $d_{RS}(\mathcal{X}_\Theta, \mathcal{X}_7^\alpha) = 0.54$, the threshold is around 0.08 which gives a maximum value of $\alpha = 0.18$ and thus a minimum number of focal elements to be used in the approximation is $M_0 = 6$.

4. COMBINING BELIEF FUNCTIONS AND MEMBERSHIP FUNCTIONS IN RANDOM SET THEORY

Here, we consider the problem of target identification using *a priori* information stored in a database containing $N = 142$ objects. Each object has a set of known characteristics such as the type, the subtype, the physical dimensions (length, width, height, RCS front, RCS top, RCS side), the minimum and the maximum cruise speeds, the maximum cruise altitude, the list of emitters, etc. Depending on the source used, the information can be best modeled in one or the other theory. In particular, an expert opinion can typically be "*the target has a small length*" and is more naturally expressed in the fuzzy set theory. However, an ESM reporting an emitter on board of the observed target is better expressed by a belief function, for example with two focal elements being (1) the set A of all objects owning this emitter, and (2) the set of all target Θ. Such a belief function traduces the fact that the ESM report can be believed to a degree $m(A)$ and that the ESM ignores that the emitter exists (with a degree $m(\Theta)$).

We use an example of an identification scenario where the three available sources are a radar, an ESM and a human agent, producing 10 pieces of information presented in the table 1.

Instant	Source of information	Information	Initial modeling theory
1	RADAR	the target is a ship	evidence theory
2	ESM	the emitter 44 is on board	evidence theory
3	Human agent	the size viewed from the top is little	fuzzy set theory
4	Human agent	the size viewed from the side is medium	fuzzy set theory
5	ESM	the emitter 77 is on board	evidence theory
6	ESM	the emitter 56 is on board	evidence theory
7	Human agent	the length of the target is small	fuzzy set theory
8	ESM	the emitter 47 is on board	evidence theory
9	ESM	the emitter 55 is on board	evidence theory
10	ESM	the emitter 103 is on board	evidence theory

Table 1: Information used in the identification scenario test.

First of all, pieces of information from the table 1 are represented in an initial theory (evidential or fuzzy set representations) using *a priori* information from the database. Information modeled in the evidence theory is represented by a couple $(0.8, 0.2)$ which means that a mass of 0.8 is associated to the set A of propositions from Θ and a mass of 0.2 is associated to the ignorance. Secondly, we transform this information into the random set formalism, using notation (8) and equations (30) and (31). The result of the fusion process after 9 combinations using Dempster's rule expressed in the random set formalism (24) is presented in figure 3. The left graphic

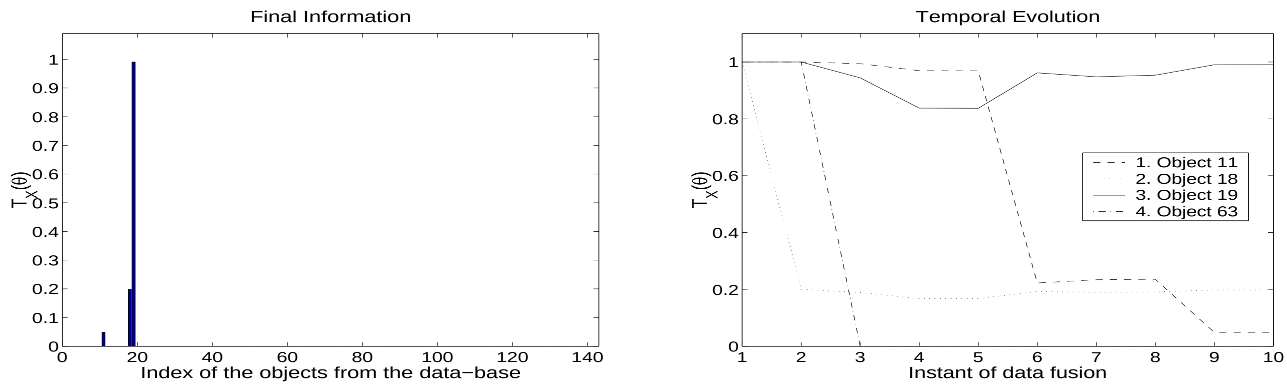

Figure 3: Target identification using random set information fusion

shows the hitting capacity of the final random set for each object (singleton) of the database. The right graphic presents the temporal evolution (during the fusion process) of the hitting capacity for the most plausible objects from the database. The object has been finally identified as the object number 19 of the database, but object 11 was the most plausible until instant 6, corresponding to an ESM report.

By introducing the approximated information into the data fusion process, the complexity of the process (computation time) is considerably decreased. Table 2 compares the different parameters of the data fusion process (the final number of focal elements, the number of flops used in the data fusion process, the distance between the approximated information and the information without any approximation). This table shows that

Approximation α	Final number of focal elements	Fusion duration (in flops)	Distance
0.000	2658	$\cong 3.5\ 10^9$	0
0.025	431	$\cong 1.4\ 10^7$	0.005
0.075	175	$\cong 1.4\ 10^6$	0.003
0.100	113	$\cong 5.0\ 10^5$	0.004
0.150	95	$\cong 3.4\ 10^5$	0.005
0.200	51	$\cong 9.2\ 10^4$	0.007
0.300	47	$\cong 7.0\ 10^4$	0.007
0.400	37	$\cong 4.7\ 10^4$	0.007
0.500	29	$\cong 2.5\ 10^4$	0.010
1.000	11	$\cong 5.0\ 10^3$	0.043

Table 2: Comparison of different parameters of the data fusion process

using $\alpha = 0.2$ for the three approximations of the membership functions (instants 3, 4 and 7), the final number of focal elements (after 9 combinations) is 51 (instead of 2658 without any approximation) and the computation time corresponds to $9.2\ 10^4$ flops (compared to $3.5\ 10^9$). Finally, figure 4 shows the hitting capacities of object 19 when different approximations for fuzzy sets are used. We note the close values between the hitting capacities obteined without approximation and using 6 focal elements (corresponding to $\alpha = 0.2$).

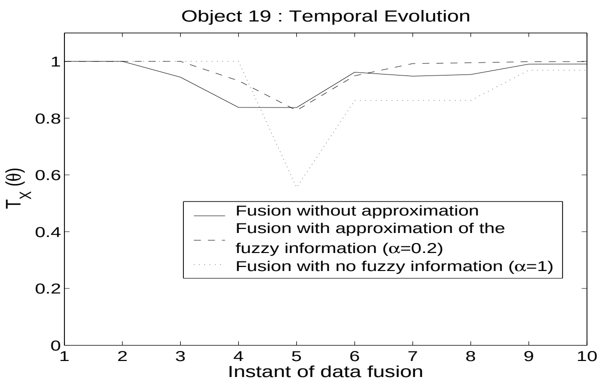

Figure 4: Hitting capacity of target 19 for different approximations of fuzzy sets

5. CONCLUSION

In this paper, we use the random set formalism to combine information coming from electronic devices and from human sources. In the first case, information is well modeled by belief functions, whereas experts opinions are better naturally modeled by fuzzy sets. While the transformation of belief functions into random set formalism is trivial, the transformation of fuzzy sets involves some discretization (using the α-cuts) and leads to a high number of focal elements (*i.e* subsets with non-null probability of occurrence). To reduce this number of subsets and avoid the explosion of the algorithm, we propose to quantify the approximation of the random set produced by the original fuzzy set by a distance between the "optimal" random set (*i.e* without approximation) and different approximations (different α-cuts). On a scenario test, we show that the number of subsets can be significantly reduced without any significant loss of performance. This research must then be extended to larger tests based on new approximations (level sets) support by some measures of performances.

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